Exploring Lattice-based Models of Relevance in Dialogue for Questions and Implicatures

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Abstract

We present work in-progress on modelling relevance in dialogue for questions and implicatures, setting out a formal programme of work on reducing the redundancy which classical logic introduces into proofs. To do this we firstly propose the use of relevance logics, then set out a lattice-theoretic generalisation of Knuth's and Hough and Purver's models of questions and answers to achieve Belnap's First-degree Entailment.

1 Introduction

Formalizing what a relevant contribution consists of in a dialogue and particularly what constitutes a relevant answer to a question is now a classical problem for formal dialogue modelling. It has enjoyed a range of existing major treatments, clearly defined as a formal challenge from Grice et al. (1975) onwards and made into a subdiscipline of pragmatics with Relevance Theory (Sperber and Wilson, 1986).

Relevance was born out of Grice's original theory of *implicature*, where speakers implicate hidden meaning which hearers can make sense of as in (1) from (Davis, 2014).

Alan:	Are you going to Paul's party?	(1)
Barb:	I have to work.	

While a literal interpretation of Barb's contribution would not permit it to be judged a relevant answer to Alan's question, the unspoken meaning that she cannot attend is recoverable. Deriving from Grice's account, as Davis (2014) notes, "Neo-Gricean theories have modified Grice's principles to some extent, and Relevance theories replace them with a principle of communicative efficiency. The problems for such principle-based theories include overgeneration, lack of determinacy, clashes, and the fact that speakers often have other goals." We add to this criticism the failure to give *real-valued* relevance measures to contributions, especially for answers to questions, though see (Hough and Purver, 2017) for one such approach in progress. In the current models the short polar answers 'yes' and 'no' would have the same degree of relevance as Barb's actual answer above, which is unintuitive.

2 Implicature with relevance logic

Here we explore some formal models of relevance agnostic to a theory of intention recognition, but which maintain the principle of least effort and maximising relevance in communication. To do this we look beyond classical logical approaches and move to a *relevance logic* approach. We furthermore explore how real-valued relevance measures of answers to questions could be incorporated into such a framework through lattice theory. We are aiming for a model which would put a real value on the degree of relevance of the contribution if certain reasoning is applied to yield the unsaid meaning and implicature.

Relevance in relevance logics is understood as ensuring every premise in a derivation is used in a proof. This has a connection in theoretical computer science to relevant type systems (Walker, 2005) and numerous engineering applications,e.g. (Cheng, 2004) or (Bruns and Huth, 2011).

In our example (1) we assume Alan and Barb would both have access to a general reasoning rule that may be available as a resource or reasoning pattern like (2).

$$\text{X is working at time } T \rightarrow \neg \text{ X can go to a party at time } T \quad \textit{(work-party exclusion rule)} \qquad (2)$$

This rule tells us when someone is working they cannot attend a party (fairly reasonable consideration for most, unless one works in e.g cater-

1. Barb can go to a party at time T $\vee\neg$ Barb can go to a party at time T $_{\{1\}}$ - *question*
2. Barb is working at time T $_{\{2\}}$ - *statement*

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3. $\neg$ Barb can go to a party at time T $_{\{2\}}$ - *instantiation of work-party exclusion rule applied to 2*
4. ResolveQuestion(1,3) $_{\{1,2\}}$ - *question resolution of question 1 by statement 3*

Figure 1: Deriving an implicated answer to a question by Relevance Logic proof.

ing, clown acts, etc.). With this rule to hand, in the spirit of (Breitholtz, 2014), we can derive a proof of the implicature that Barb cannot go to the party at that time which can resolve Alan's question, as shown in Fig. 1. We use Mares (2004)'s logical notation where the curly brackets containing the indices of the premises used in that line. The proof in Fig. 1 shows how both premises are used to derive the conclusion in line 4, which itself uses the implicature in line 3.

While this seems better than a classical logic approach because redundancy is minimized, the problem remains that we still don't have a handle on a real-valued relevance which could lead to a computational model of selecting relevant rules.

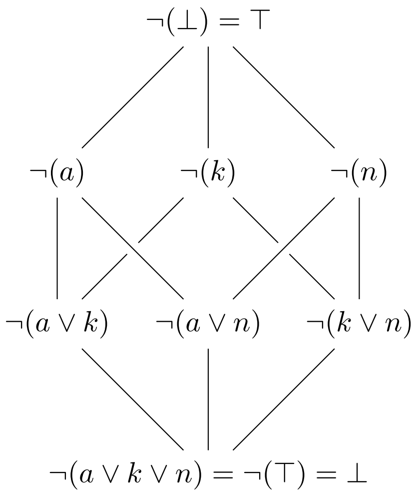

Figure 2: De Morgan lattice

3 Towards Relevance Logic Lattices for Real-valued Relevance

To model the real-valued relevance of answers to questions and implicatures, we look to work by Knuth (2005) and (Hough and Purver, 2017) whereby a boolean algebra statement lattice like that in Fig. 3 in the Appendix allows real-valued probabilities to be assigned to the atoms of the lattice and then consequently to the joins of those elements. Questions are derived from this lattice as the joins of all the downsets of these elements. In such a framework in our example in Fig. 1, a relevance value is contingent on the the real-valued inclusion of statement 3 in statement 2 on the lattice after the application of the 'work-party exclusion rule'– if this is sufficiently high, we could rule this a relevant application of the general rule in order to derive 4.

While this seems to give us what we want, a problem of relevance remains, but this time in terms of the available answers to questions: in Knuth's analysis, all questions can trivially evaluate to $\bot$. In fact $\top$ in Knuth's analysis is co-extensive with the entire space of questions and answers, which is counter-intuitive for any question with any content that does not involve asking whether something is true or false. We propose adopting a different underlying algebra which helps block these issues, and seems to capably model relevance both as a conversational implicature and as a logical consequence relation. We believe this can be achieved through a De Morgan lattice like Fig. 2 where the trivial results can be minimized and we can achieve a Relevant logic known as First-Degree Entailment (FDE). (Belnap, 1977) and their collaborators (Anderson et al., 2017) show how this can be achieved– see Fig. 4 Appendix for an illustration.

4 Future Work

In future work we would like to leverage the power of Knuth's work on probability and information theory with question and statement lattices and the De Morgan lattices described above for deriving a real-valued relevance of a contribution resolving the central issue. We have evidence that Knuth's approach can be generalised and De Morgan algebras are, in addition to being the backbone of FDE described above, investigated in fuzzy logic circles– for example forming an adequate algebraic semantics for a Lukasiewicz logic (Nguyen and Walker, 1996).

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Appendix

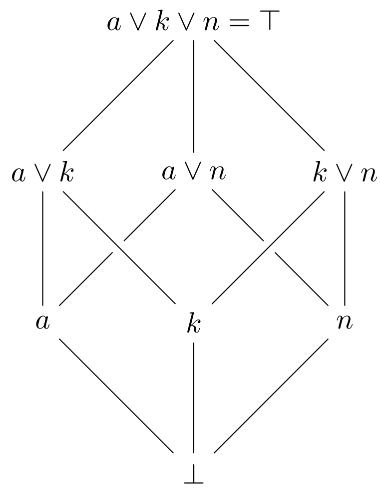

Figure 3: A Knuth-style lattice of statements for a Boolean algebra

$$\frac{\Gamma \vdash \phi \qquad \Gamma \vdash \psi}{\Gamma \vdash \phi \wedge \psi} \wedge \mathrm{I}$$

$$\frac{\Gamma \vdash \phi_i}{\Gamma \vdash \phi_1 \vee \phi_2} \vee \mathrm{I} \quad (i \in \{1,2\})$$

$$\frac{\Gamma \vdash \phi_1 \wedge \phi_2}{\Gamma \vdash \phi_i} \wedge \mathrm{E} \quad (i \in \{1,2\})$$

$$\frac{\Gamma \vdash \phi \vee \psi \qquad \Delta, \phi \vdash \chi \qquad \Delta, \psi \vdash \chi}{\Gamma, \Delta \vdash \chi} \vee \mathrm{E}$$

$$\frac{\Gamma \vdash \neg\neg\phi}{\Gamma \vdash \phi} \neg\neg\, \mathrm{E}$$

$$\frac{\Gamma \vdash \phi}{\Gamma \vdash \neg\neg\phi} \neg\neg\, \mathrm{I}$$

$$\frac{\Gamma \vdash \neg(\phi \vee \psi)}{\Gamma \vdash \neg\phi \wedge \neg\psi} \text{DeMorgan(i)}$$

$$\frac{\Gamma \vdash \neg(\phi \wedge \psi)}{\Gamma \vdash \neg\phi \vee \neg\psi} \text{DeMorgan(ii)}$$

Figure 4: FDE