Questions as Functions

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Abstract

Most previous studies on questions pay much attention to the representation and answerhood problems, but are less concerned with the way how they are derived in discourse. Taking Martin-Löf's constructive type theory as the starting point, this article develops a function-based theory of question, which provides us with new formal tools for representing the various kinds of natural language questions and modeling their inferential behavior in a unified way.

1 Introduction

Questions are undoubtedly important in both logic and linguistics. Studies on questions in formal logic date back to the late twenties of the last century. Since then, many different accounts have emerged as a result of the development of logical tools. However, while much attention has been paid to the representation of questions in a particular formalism or the definition of questions in terms of answerhood (see a detailed review in Wiśniewski, 2015), less is devoted to exploring the way a particular question is derived in discourse context. Taking Martin-Löf's (1984) constructive type theory (CTT, henceforth) as the starting point, this article proposes to analyze questions as different kinds of functions and provides a theory for their derivation (especially, question-evocation) in discourse.

The article is organized as follows: section 2 introduces a linguistic taxonomy of questions in natural language, mainly based on Wiśniewski's (2013) and Ginzburg's (2012) informal analyses and thus sets a list of desiderata for the theory to be developed in subsequent sections; section 3 is a brief reminder of Martin-Löf's constructive type theory; section 4 compares (formal) questions to functions, and distinguishes between three different kinds of erotetic functions, namely, proof search, type inference, and type checking; section 5 applies the theory to representing the different kinds of natural language questions that we introduce in section 2, and also provides a preliminary analysis of erotetic reasoning in dialogue; section 6 briefly compares the new theory with other previous proposals within the same approach; and the final section concludes the article and outlines the tracks for future research.

2 A Taxonomy of Questions in Natural Language

The method to classify natural language questions varies depending on the criteria one takes into account. In English, one may easily distinguish between a yes-no question and a wh-question, as they are obviously different in their syntactic form. However, to understand and to model the inferential behavior of questions, we are more interested in the meaning part of questions. Alternative Semantics is usually conceived as the standard theory for the meaning of questions (Hamblin, 1973), according to which, the semantic meaning of a question is the set of alternative propositions that answer that question. The size of the answer set (or the (in)finiteness of answer alternatives) is taken by some researchers as an essential criterion for classifying questions. For instance, Wiśniewski (2013) made the following classification:

(1) **Open-condition question**: An open-condition question expresses open conditions requested to be filled. For example: Who left for London?

(2) **Delimited-condition question**: A delimited-condition question expresses a condition to be filled, yet associated with a list of instances. For example: Who left for London, John or Mary?

(3) **Choice question**: A choice question lists certain implicit alternatives among which a choice is requested to be made. For example: Did John leave for London?

(4) **Topically-oriented question**: A topically-oriented question, for instance, a why-question, expresses a condition of the form *ρ because* ... requested to be filled. For example: Why did John leave for London?

The distinction between (1) and (2) is evident as they correspond respectively to a restricted and unrestricted set of alternative answers. (3) can also be put in the category of delimited-condition question, but it differs from (2) in the sense that it is a request for a truth value that can be assigned to the encoded proposition. (4) is different from all of the above cases: it contains a complete proposition, which is already presupposed to be true, and it seeks an assumption under which the truth of the proposition holds. In logic, this is equivalent to conducting an abductive research, i.e., seeking a possible explanation for a given confirmed conclusion. For the convenience of discussion, we will call it *abductive research* hereafter. [1]

The use of questions (1-4) is not homogenous in discourse: while in some cases, they can be uttered out-of-blue, in some others, they are used as a response to a preceding move. Drawing upon Ginzburg's (2012) analysis of (meta)communicative questions, we distinguish three main kinds of question use:

(5) **Simple query**: A simple query is a question that can (but need not) be introduced out-of-blue, and imposes no specific expectation towards the answer.

(6) **Truth confirmation**: A truth confirmation question arises as a response to the previously asserted proposition and requests a confirmation of the truth value assigned to that proposition. For example:
  a. A: John left for London.
  b. B: Did John leave for London? (= Are you sure that John left for London?)

(7) **Clarification**: A clarification question arises as a response to the previously asserted proposition and requests a clarification of either the clausal content or the intended content of a given (sub-)utterance.
  (7.1) **Clausal confirmation**: A clausal confirmation queries the semantic contribution of a particular constituent. For example:
    a. Emily: John left for London.
    b. David: John? (= Are you saying John?)
  (7.2) **Intended content**: An intended content clarification queries the content associated with a given (sub-)utterance. For example:
    a. Emily: John left for London.
    b. David: John? (= Who is John?)

A successful theory of question is expected to be able to represent the abovementioned different types of natural language questions, and to explain how these questions are inferred and resolved in discourse context. The proposal is based on Martin-Löf's CTT, which is to be briefly introduced in the next section.

3 A Brief Reminder of Constructive Type Theory

Constructive type theory (CTT) is a logical framework developed in a series of papers published by Per Martin-Löf since the late 70s. Central to CTT is the principle of propositions as types, according to which, a proposition (or a formula) can be interpreted as a set whose elements count as the proofs for that proposition. The most fundamental notion of CTT is that of a judgment: a judgment $a : A$ classifies a proof object a as being of a specified type A. It can be read in a number of different ways, for instance, a is an element of the set A, or a is a proof of the proposition A (Martin-Löf 1984: 4). If a proposition has a proof, it is true. The law of excluded middle $A \vee \neg A$ thus does not hold in CTT.

[1] However, it is not to say that abductive research (i.e., a why-question) is the only kind of topically-oriented question. In Wiśniewski's (2013) examples, we also find another kind – how-questions – which usually has a procedure-seeking function, that is, questioners making such inquiries are looking for an explanation of the procedure (instead of the reason) of doing something. For a detailed discussion of the difference between abductive research and procedure-seeking questions, see, for instance, a logical analysis in Wang (2018).

There are in general two kinds of judgments in CTT, namely, categorical judgments and hypothetical judgments. A categorical judgment does not depend on any assumptions. There are four different types of categorical judgment (Martin-Löf 1984: 3):

$$A : type \qquad\qquad A = B : type$$
$$a : A \qquad\qquad a = b : A,$$

Hypothetical judgments are those that are dependent on a set of assumptions. There are two basic forms of hypothetical judgment in CTT (Martin-Löf 1984: 9-10):

$$B : type\ (x : A) \qquad\qquad b(x) : B\ (x : A).$$

If one takes the antecedent $x : A$ as the domain and the consequent $B : type$ and $b(x) : B$ as the range, the two hypothetical judgments can also be understood as introducing two corresponding functions:

$$f\!: (x : A) \rightarrow B(x) : type \qquad\qquad f\!: (x : A) \rightarrow b(x) : B(x).$$

A hypothetical judgment can take an infinite number of assumptions, which constitute the *context* for making that judgment:

$$b : B\ (\Gamma) \text{ or } \Gamma \vdash b : B, \text{ where } \Gamma : context$$

For the time being, a brief introduction to CTT shall be sufficient. For more technical details, see, for instance, Martin-Löf (1984), Ranta (1994), and Granström (2011).

4 Questions as Functions

4.1 Erotetic Judgment

The relationship between assertion (as a kind of speech act) and judgment has been discussed extensively in logicians' and philosophers' studies. The most classical view, due to Frege (1918), takes assertion as the outward sign of a judgment. Similar ideas are found in Dummett (1973) and Granström (2011), among many others (see van der Schaar 2011 for a detailed review). In terms of CTT, by making an assertion, i.e., a typed judgment $a : A$, one takes a public commitment to providing a justification for A. Kvernenes (2017) generalizes the analysis to explaining questions. According to Kvernenes (2017), questions should also be considered as some sort of judgments, as they behave like assertions in two important aspects: on the one hand, asking a question stands for one's commitment to its answerability, i.e., there exists an answer that can resolve the question (c.f., the existential presupposition of questions, see Dayal 2017 for a review); on the other, the questioner is also believed to able to justify the inquiry he makes. In line with Kvernenes (2017), a distinction is made between *assertive judgments* ($\vdash J$) (*evident judgments* in terms of Kvernenes) and *erotetic judgments* ($?J$) (*demanding judgments*, ibid; cf. Wiśniewski's *e-formula*):

(8) **Assertive judgment**: An assertive judgment is the result of the act of making an assertion ($\vdash J$), that is, to take a public commitment to providing justification for the truth of the proposition.
(9) **Erotetic judgment**: An erotetic judgment is the result of the act of asking a question ($?J$), that is, to take a public commitment to providing justification for the answerability of the question (or equivalently, for the truth of the existential closure of that question).

The Moorean paradoxes of assertion and question (see van der Schaar 2011 and Wall 2012 for more comments) show some hints on the condition under which one is entitled to assert and query. Consider the examples in (10) and (11):

(10) Moorean paradoxes of assertion
  a. #It rains, but I doubt it.

b. #It rains, but I don't believe it.
c. #It rains, but I have no evidence for it.

(11) Moorean paradoxes of question
a. #Who killed John? – but I doubt that anyone killed him.
b. #Who killed John? – but I don't believe that anyone killed him.
c. #Who killed John? – but I don't have any evidence for that someone killed him.

The oddity of the sentences in (10) suggests that one is entitled to assert if and only if he can justify the proposition that he believes to be true; whereas the sentences in (11) imply that one is entitled to query if and only if he can justify the question that he believes to be answerable.

4.2 Representing Questions with Different Functions

Ginzburg (2012) expresses an idea, which is akin to ours: a question can be considered as a propositional abstract, that is, a function from records (i.e., proof objects for record-types) into propositions. In what follows, we further exploit this idea and proposes to categorize the erotetic function into three different kinds: proof-search, type-inference, and type-checking. [1]

In order to make an assertion $\vdash a : A$ where $A : prop$, one needs to know at least three kinds of information: the type (either simple or complex), the proof (or at least the existence of a proof), and the coherence of the proposition with other true propositions in the system. When only some of them are available, one may put forward a request for the lacked pieces. Therefore, we may identify three different kinds of inquiries:

(12) **Proof search:** Given a type *A*, search for a proof object *x* such that *x* can be classified as being of type *A* under the context Γ:

$$\Gamma\ ?_{\mathrm{P}}\ x : A\ [b(x) : B] = \Gamma\ ?_{\mathrm{P}}\ (\lambda x)b(x) : (x : A)[B]$$

which can be interpreted as a question that queries the (existence of a) proof object *x* of the type *A* such that $b(x^A) : B$.

(13) **Type inference**: Given a proof object *a*, search for a type *x* such that *a* can be classified as being of the type *x* under the context Γ:

$$\Gamma\ ?_{\mathrm{T}}\ a : x\ [b(a^x) : B] = \Gamma\ ?_{\mathrm{T}}\ (\lambda x)b(a^x) : ((x : type)[a : x])[B]$$

which can be interpreted as a question that queries the type *x* of the proof object *a* such that $b(a^x) : B$.

(14) **Type checking**: Given a proof object *a* and a type *A*, decide whether *a* is of type *A* under the context Γ:

$$\Gamma\ ?_{\mathrm{C}}\ a : A[b(a^A) : B] = \Gamma\ ?_{\mathrm{C}}\ (\lambda x)b(a^x) : ((x : type)[a : A(x)])[B]$$

which can be interpreted as a question that queries the correctness of the judgment $a : A$ such that $b(a^A) : B$.

Making a type checking usually follows two steps: (i) starting with a given proof object *a*, infer the corresponding type *x*, and then (ii) compare the inferred type *x* with the given type *A* and return the result $\Gamma \vdash a : A$ if A=x, or otherwise, return an error, i.e., $\Gamma \vdash a :/ A$ (if $A \neq x$). Type checking thus also comprises *proof checking* in virtue of their symmetrical relation: to check whether a proof object is of a particular type is also to check whether the type can categorize the given proof.

4.3 Logical Rules for Question-Evocation

In the previous section, we discussed the possible formal questions that are allowed in CTT. The remaining question is how these questions are derived in discourse context. In his pioneering research, Wiśniewski (2013) makes a distinction between two different but interrelated derivational processes of questions: (i) question-evocation, in which a question is evoked based on a list of assertions or possible assertions, and (ii) erotetic implication, in which a question is implied by another question. Due to space limitations, this section will only concentrate on the first kind, the question-evocation.

[1] The terms are borrowed from computer science. See Ranta (2012) for a technical explanation.

Let's start with an example:

(15) Context: Emily's cousin John left for London last week.
  a. Emily: John left for London.
  b. David: Who is John?
  c. Emily: My cousin.

The question (15b), as an intended content clarification, arises as a result of two premises, i.e., that David does not know who John is, and that Emily knows that because she has mentioned the name in (15a). In this case, both premises are assertions (either explicit made or stored as part of the implicit world knowledge), and the derivation of the question (15b) thus belongs to the category of question-evocation. (15c) resolves the question (15b) and thus favors David's grounding of (15a).

The question derivation and resolution can be modeled by using a series of natural deduction rules, namely, formation rules, introduction rules, elimination rules, and equality rules:

(16) Proof search ($?_\mathrm{P}$)

$$\frac{\begin{array}{c}(\Xi, (\Gamma \vdash x : A)) \\ \Gamma \vdash b(x^A) : B\end{array}}{\Xi\ ?_\mathrm{P}\ (x : A)[b(x^A) : B] : \mathit{function}}\ ?_\mathrm{P}F$$

$$\frac{\begin{array}{c}(\Xi, (\Gamma \vdash x : A)) \\ \Gamma \vdash b(x^A) : B\end{array}}{\Xi\ ?_\mathrm{P}\ (\lambda x)b(x^A) : (x : A)[B]}\ ?_\mathrm{P}I$$

$$\frac{\begin{array}{c}\Xi\ ?_\mathrm{P}\ p : (x : A)[B] \\ (\Gamma \vdash a : A)\end{array}}{\Gamma \vdash ap(p, a) : B(a : A)}\ ?_\mathrm{P}E$$

$$\frac{\begin{array}{c}(\Xi, (\Gamma \vdash x : A)) \\ \Gamma \vdash b(x^A) : B \qquad (\Gamma \vdash a : A)\end{array}}{ap((\lambda x)b(x^A), a) = b(a/x^A) : B}\ ?_\mathrm{P}Eq$$

(17) Type inference ($?_\mathrm{T}$)

$$\frac{\begin{array}{c}(\Xi, (\Gamma \vdash a : x\ (x : \mathit{type}))) \\ \Gamma \vdash b(a^x) : B\end{array}}{\Xi\ ?_\mathrm{T}\ ((x : \mathit{type})[a : x])[b(a^x) : B] : \mathit{function}}\ ?_\mathrm{T}F$$

$$\frac{\begin{array}{c}(\Xi, (\Gamma \vdash a : x\ (x : \mathit{type}))) \\ \Gamma \vdash b(a^x) : B\end{array}}{\Xi\ ?_\mathrm{T}\ (\lambda x)b(a^x) : ((x : \mathit{type})[a : x])[B]}\ ?_\mathrm{T}I$$

$$\frac{\begin{array}{c}\Xi\ ?_\mathrm{T}\ p : ((x : \mathit{type})[a : x])\ [B] \\ (\Gamma \vdash A : \mathit{type})\end{array}}{\Gamma \vdash ap(p, A) : B(a : A)}\ ?_\mathrm{T}E$$

$$\frac{\begin{array}{c}(\Xi, (\Gamma \vdash x : \mathit{type})) \\ \Gamma \vdash b(a^x) : B \qquad (\Gamma \vdash A : \mathit{type})\end{array}}{ap((\lambda x)b(a^x), A) = b(a^{A/x}) : B}\ ?_\mathrm{T}Eq$$

(18) Type checking ($?_\mathrm{C}$)

$$\frac{\begin{array}{c}(\Xi, (\Gamma \vdash a : A(x)\ (a : x\ (x : \mathit{type})))) \\ \Gamma \vdash b(a^x) : B\end{array}}{\Xi\ ?_\mathrm{C}\ (((x : \mathit{type})[a : x])[a : A(x)])[b(a^x) : B] : \mathit{function}}\ ?_\mathrm{C}F$$

$$\frac{\begin{array}{c}(\Xi, (\Gamma \vdash a : A(x)\ (a : x\ (x : \mathit{type})))) \\ \Gamma \vdash b(a^x) : B\end{array}}{\Xi\ ?_\mathrm{C}\ (\lambda x)b(a^x) : (((x : \mathit{type})[a : x])[a : A(x)])[B]}\ ?_\mathrm{C}I$$

$$\frac{\begin{array}{c}\Xi\ ?_\mathrm{C}\ p : (((x : \mathit{type})[a : x])[a : A(x)])\ [B] \\ (\Gamma \vdash A : \mathit{type})\end{array}}{\Gamma \vdash ap(p, A) : B(a : A)}\ ?_\mathrm{C}E$$

$$\frac{\begin{array}{l}(\Xi, (\Gamma \vdash x\ :type)) \\ \Gamma \vdash b(a^x) : B \qquad (\Gamma \vdash A\ :type)\end{array}}{ap((\lambda x)b(a^x), A) = b(a^{A/x}) : B}\ ?_C Eq$$

The formation rule (F) states how a formal question is formed. The introduction rule (I) describes how a particular type of formal question is introduced. The elimination rule (E) suggests how the interrogative operator is eliminated or equivalently, how the question is answered, whereas the equality rule (Eq) justifies the elimination rule by stating how it operates on the canonical elements that are formed by the introduction rule. Since we are only interested in question-evocation, all of the premises in the derivation process are assertions. In the next section, all of these formal tools are implemented to model natural language questions.

5 Representing Natural Language Questions

In section 2, we set a list of desiderata for a formal theory of question: it must be able to represent both the form and the derivation of the different kinds of natural language questions: open-condition questions, closed-condition questions (i.e., delimited-condition questions and choice questions), abductive research, and also the three kinds of pragmatic use of questions in discourse context: simply query, confirmation, and clarification (i.e., clausal content confirmation and intended content confirmation). In what follows, we will show how this is done by using the theory we proposed in section 4.

First of all, as we already mentioned in the above discussion, choosing a particular form of question for making a query depends on what information is available and what is absent in discourse context. If both proof and type are present, the questioner can check the correctness of the judgment; whereas when only some of them are available, one may put forward a request for the lacked pieces. Now let's consider in turn what kinds of information are absent (and thus need to be requested) in the various questions we have seen in section 2.

Consider for instance the open-condition and closed-condition questions in (19).

(19) Open-condition and closed-condition questions
  a. Who left for London?
  b. Who left for London, John or Mary?
  c. Did John leave for London?

In both (19a) and (19b), the questioner presupposes that someone left for London and would like to know who the guy is. In type-theoretical terms, the propositional type is available and what is requested is an explicit proof object. Delimitating the conditions does not influence the question-evocation process but constrains the possible subquestions to be generated. Splitting a question into subones reflects a process of question-implication, which is beyond the scope of this article. Both (19a) and (19b) can be modeled by using the proof search operation:

(20) a. $\Gamma\ ?_P\ (\lambda x)a(x) : (x : human)[A]$.
  b. $\Gamma\ ?_P\ (\lambda x)a(x) : (x : human)[A]$, where only $j : human$ and $m : human$.
  Notation: Here and in what follows, A=John left for London.

In the case of (19c), the speaker asks for a truth value (or Boolean value) that can be assigned to the proposition and thus is equivalent to searching for either a positive proof or a negative proof (but not both) for the corresponding form of the encoded proposition. As a result, it can be represented in two different ways: either as a type inference question (21a), or as a proof search question (21b).

(21) a. $\Gamma\ ?_T\ (\lambda x)V(A) : (x : bool)[x]$, where V is a valuation function that assigns a truth value to each proposition.
  b. $\Gamma\ ?_P\ (\lambda x)x : A \mid \neg A$, where | is a symbol for exclusive disjunction.

In an abductive research, such as (22), one would like to know how a given judgment – that John left for London – as the conclusion, is arrived at. By doing so, one takes for granted that the speaker who made that judgment/assertion must be able to justify his words, that is, to provide other true propositions, such as (22b), as premises. As a result, an abductive research can be modeled by using the type inference operation, as given in (23).

(22) Abductive research
   a. David: Why did John leave for London?
   b. Emily: He found a position in HSBC.

(23) $\Gamma\ ?_{\mathrm{T}}\ (\lambda x)a(b^{x}) : ((x : prop)[b : x])[A]$.

The different pragmatic uses of questions can also have a proper explanation and formalization by using the new formal tools. Consider first a truth confirmation question in (24).

(24) Truth confirmation
   a. Emily: John left for London.
   b. David: John? (= Are you sure it was John?)

David was told by Emily that John left for London but by making a truth confirmation question (24b), it is obvious that David is still hesitating to accept the truth of the proposition. What David intends to do by asking (24b) is to check again whether the statement is true, or in other words, to get Mary to think over her words. In type-theoretical terms, what David seeks to check is not the truth, but the correctness of the typed judgment, i.e., whether the proof object is correctly typed. Consequently, it can be modeled by using the type checking operation:

(25) $\Gamma\ ?_{\mathrm{C}}\ (\lambda x)a(b^{x}) : ((x : type)[b : j(x)])[A]$.

Clarifications are different from truth confirmations. The following examples in (26) illustrate the two kinds of clarification questions.

(26) Emily: John left for London.
   a. Intended content
      David: John? (= Who is John?)
   b. Clausal confirmation
      David: John? (= Are you saying John?)

(26a) is an intended content clarification, by which the speaker David requests the semantic content associated with the constituent 'John'. After being told that John left for London, David knows that there exists some guy whose name is John and who had left for London, but he cannot make it cohere with his world knowledge as he doesn't know who John is. In type-theoretical terms, the proof is already given and what needs to be requested is the corresponding type that categorizes that proof. Therefore, it can be considered as a type inference question:

(27) $\Gamma\ ?_{\mathrm{T}}\ (\lambda x)a(j^{x}) : ((x : type)[j : x])[A]$

In the case of (26b) – a clausal confirmation question –, the speaker David knows that Emily has asserted something and she holds a proof for her words, but David does not know exactly the semantic content of the constituent 'John' inside Emily's assertion. It could be due to that David only overheard Emily's words or that David was not being attentive while Emily was speaking. Consequently, by (26b), David is making a type inference, as given in (28), but it differs from an ordinary type inference in that the speaker already provides an alternative option, that is, a possible type 'John'.

(28) $\Gamma\ ?_{\mathrm{T}}\ (\lambda x)a(b^{x}) : (((x : type)[b : x])[b : j(x)])[A]$

To sum up, each kind of natural language question that we introduced in section 2 can be represented by a specific erotetic function, concretely,

Function	Erotetic judgment	Natural language question
Proof search	$\Gamma\ ?_{\mathrm{P}}\ (\lambda x)b(x) : (x : A)[B]$	• Open-condition question • Closed-condition question
Type inference	$\Gamma\ ?_{\mathrm{T}}\ (\lambda x)b(a^{x}) : ((x : type)[a : x])[B]$	• Abductive research • Clausal confirmation • Intended content
Type checking	$\Gamma\ ?_{\mathrm{C}}\ (\lambda x)b(a^{x}) : ((x : type)[a : A(x)])[B]$	• Truth confirmation

Finally, consider a dialogue fragment in (29), which can be represented in the form of the deduction-like tree, as given in (30). In such a way, the inferential relationship between every two dialogue moves is clearly exhibited.

(29) Context: Emily's cousin John found a position in HSBC and left for London last week.
  a. Emily: John left for London.
  b. David: Who is John?
  c. Emily: My cousin.
  d. David: Okay, why did he leave for London?
  e. Emily: He found a position in HSBC.
  f. David: Are you sure?
  g. Emily: Yes.

(30) **Notation**: Γ_0 stands for the original context of Emily and Ξ_0 for that of David. A=John left for London, B=Emily's-cousin(x), C=John found a position in HSBC.

(28a)
$$\Gamma_0 \vdash a : A$$
$$\Downarrow$$
$$\Gamma_0 \vdash a(j^{x}) : A$$

(28b)–(28c)
$$\dfrac{\dfrac{(\Xi_0, (\Gamma_0 \vdash j : x\ (x : type)))}{\Xi_1\ ?_{\mathrm{T}}(\lambda x)a(j^{x}) : ((x : type)[j : x])[A]}\ ?_{\mathrm{T}}I \qquad (\Gamma_1 \vdash j : B)}{\Gamma_1 \vdash a(j^{B/x}) : A}\ ?_{\mathrm{T}}E$$
$$\Downarrow$$
$$\Gamma_1 \vdash a(c^{x}) : A$$

(28d)–(28e)
$$\dfrac{\dfrac{(\Xi_1, (\Gamma_1 \vdash c : x\ (x : type)))}{\Xi_2\ ?_{\mathrm{T}}(\lambda x)a(c^{x}) : ((x : type)[c : x])[A]}\ ?_{\mathrm{T}}I \qquad (\Gamma_2 \vdash c : C)}{\mathit{\Gamma}_2 \vdash a(c^{C/x}) : A}\ ?_{\mathrm{T}}E$$

(28f)–(28g)
$$\dfrac{\dfrac{(\Xi_2, (\Gamma_2 \vdash c : C(x)\ (c : x\ (x : type))))}{\Xi_3\ ?_{\mathrm{T}}(\lambda x)a(c^{x}) : (((x : type)[c : x])[c : C(x)])[A]}\ ?_{\mathrm{C}}I \qquad (\Gamma_3 \vdash c : C)}{\Gamma_3 \vdash a(c^{C=x}) : A}\ ?_{\mathrm{C}}I$$

6 Comparison with Other CTT-based Formalisms

As we have mentioned in section 4.2, it is not a new idea to analyze questions as functions. Indeed, we have benefitted a lot from three early attempts within the same approach: a preliminary type-theoretical analysis of question by Ranta (1994), the propositional abstract account in Ginzburg and Sag (2000) and Ginzburg (2012), and a CTT-based logical analysis of inquiries by Kvernenes (2017).

Ranta (1994) is probably the first one who attempts to provide a new paradigm for natural language research from a constructive type-theoretical perspective. With regard to questions, Ranta (1994) makes a dichotomy between propositional questions, in which (at least) two alternative propositions are given as choices, and wh-binding questions, in which a wh-operator binds the variable x and determines its semantic range.

(31) $A \mid B$, where $A : prop$ and $B : prop$.
(32) $(\mathrm{Wh}\ x : A)B(x)$, where $A : prop$ and $B(x) : prop\ (x : A)$.

Ranta's (1994) analysis is inspiring but also rudimentary. Some types of questions, such as the polar and wh-constituent questions can be well modeled following his suggestions, whereas some others, especially, the different pragmatic uses of questions are beyond his concerns. Moreover, Ranta (2014) cares less about the epistemic basis of querying (i.e., what is already known and what needs to be requested), nor the conditions under which a question is inferred.

Ginzburg's (2012) function-based analysis conceives a question as a propositional abstract (similar to an abstract in lambda calculus) – a function from record-level proofs to a complete proposition. This naturally applies to the representation of wh-constituent questions, where a variable x is bound by a wh-expression that specifies the record-type (for instance, x : *person*, a type determined by the wh-expression *who*). To answer a wh-constituent question, accordingly, is to locate a proof object that can be classified as being of the specified type. Polar questions are treated in Ginzburg (2012) (and also Ginzburg and Sag 2000) as 0-ary abstracts/types. Though technically sound, the notion of 0-ary abstract/type is somewhat dissatisfactory in the sense that it is not epistemically underpinned. The alternative way we suggest in section 5 is motivated by the core idea of CTT, that the truth of a proposition is a side product of provability, and to ask a polar question is therefore equivalent to checking whether there exists a proof or a counterproof for that proposition.

A recent attempt to interpret questions in CTT is made by Kvernenes (2017), according to whom, three types of formal inquiries need to be differentiated: (i) *type declaration inquiry* $?_{\text{type}} A\ [J(x)]$, which searches for the type of an assumption of $J(x)$; (ii) *assumption inquiry* $?_{\text{ass}}\, x : A\ [J(x)]$, which requests an assumption of $J(x)$; and (iii) *definition inquiry* $?_{\text{def}}\, x : A\ [J(x)]$, which seeks a proof object (p.p. 54-57). Leaving aside the syntactic difference, Kvernenes' (2017) proposal comes closest in concept to ours: making a type declaration inquiry is actually doing a type inference, whereas seeking a definition is equal to searching for a proof. However, a remarkable difference is noteworthy: the assumption inquiry, which Kvernenes (2017) treats as an independent type, is subsumed as a special kind of type inference question in our system. The reason is, as we have explained in section 3.2, to ask for an assumption, the questioner actually presupposes that the speaker can provide such a reason and what he actually asks for is not the truth of the reason, but the propositional content. Another difference between the two proposals is that we include an additional formal question type – type checking – which is absent and cannot be subsumed under any of the three inquiry types in Kvernenes' (2017) system.

7 Conclusion and Future Research

Drawing upon Martin-Löf's constructive type theory, this article develops a function-based theory of question, which provides us with new formal tools for representing the various kinds of natural language questions and modeling their inferential behavior in discourse context. Yet, the theory is far from being satisfactory, as there are still plenty of questions and phenomena that require a close examination in the future. First of all, as we have already mentioned, a question can not only be inferred from a list of assertive premises, but it can also be implied by another question. The latter kind of question derivation – what Wiśniewski (2013) calls erotetic implication – needs to be analyzed in more detail and compared with the former kind. Second, there are many different non-canonical questions in human language, for instance, declarative questions, echo questions, tag questions, and rhetorical questions, all of which deserves close attention and future consideration. Finally, as suggested by an anonymous reviewer, it would be an interesting question how the above analysis may go beyond a technical application toward a deeper understanding of the notion of question: what counts as a question, why would one ask a question, and what are the norms that should be observed while asking a question, etc.

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References

Veneeta Dayal. (2017). *Questions*. Oxford University Press, Oxford.

Michael Dummett. (1973). *Frege: Philosophy of language*. Harvard University Press, Cambridge, Massachusetts.

Gottlob Frege. 1918. Der Gedanke. Eine Logische Untersuchung In *Beiträge zur Philosophie des deutschen Idealismus*, I (1918–1919): 58–77. Translated as 'Thoughts', by P. Geach and R. Stoothoff, in McGuinness (ed.) 1984, *op. cit.*, pp. 351–372.

Jonathan Ginzburg. 2012. *The Interactive Stance, Meaning for Conversation.* Oxford University Press, Oxford.

Jonathan Ginzburg and Ivan Sag. 2000. *Interrogative investigations*. CSLI publications, Stanford.

Johan G. Granström. 2011. *Treatise on Intuitionistic Type Theory.* Springer, Berlin.

Charles L. Hamblin, 1973. Questions in Montague English. *Foundations of Language*, 10: 41-53.

Per Martin-Löf 1984. *Intuitionistic Type Theory*. Bibliopolis, Napoli.

Hans C. N. Kvernenes 2017. *Research Agendas, Inquiries and Answers in Constructive Type Theories*. Master thesis. Bergen: Universitetet i Bergen. Retrieved from http://bora.uib.no/bitstream/handle/1956/16330/Master-oppgave.pdf?sequence=1.

Aarne Ranta. 1994. *Type-Theoretical Grammar*. Oxford University Press, Oxford.

Aarne Ranta. 2012. *Implementing Programming Languages. An Introduction to Compilers and Interpreters*. College Publications, London.

Maria van der Schaar. 2011. Assertion and Grounding: A Theory of Assertion for Constructive Type Theory. *Synthese*, 183(2), 187-210.

David Wall. 2012. A Moorean paradox of desire. *Philosophical Explorations*, *15*(1): 63-84.

Andrzej Wiśniewski. 2013. *The Posing of Questions: Logical Foundations of Erotetic Inferences.* Springer, Berlin.

Andrzej Wiśniewski. 2015. Semantics of Questions. In S., Lappin., and C., Fox. (eds.), *The Handbook of Contemporary Semantic Theory*, 2nd Edition. Wiley-Blackwell, Oxford.